Jackson 3.3
scratch

x^3 $3x^2$ $6x$ $\boxed{6.}$

$\Phi(\theta = \frac{\pi}{2}, r = R, \phi) = V.$

$= \Phi = \sum_{n=0}^{\infty} B_l r^{-l} P_l[\cos\theta]$

$= \sum_{l=0}^{\infty} B_l r^{-(l+1)} \Big|_{r=b} P_l[\cos\theta] = V.$

$\sum_{l=0}^{\infty} B_l b^{-(l+1)} P_l[\cos\theta] = V.$

$\sum_{l=0}^{\infty} B_l R^{-(l+1)} P_l[0] = V.$

$\int \Phi P_l[\cos\theta'] \sin\theta' d\theta = \frac{2}{2l+1} B_l R^{-(l+1)}$

$B_l = \frac{2l+1}{2} R^{l+1} \int \Phi P_l[\cos\theta'] \sin\theta' d\theta$

$B_l = \frac{2l+1}{2} R^{l+1} \frac{V}{\pi} P_l[0]$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$P_l = \frac{1}{2^l l!} \frac{d^l}{dx^l}(x^2-1)^l$ $P_l \Phi$

$(x^2-1)^l = \sum_{n=0}^{l} \binom{l}{n} (x^2)^n (-1)^{l-n}$

$P_l[0] = \begin{cases} \frac{1}{2^l l!} l! \binom{l}{l/2} (-1)^{l/2} & l \text{ even} \\ 0 & l \text{ odd} \end{cases}$ we need the x^l term

$= \frac{1}{2^l \cancel{l!}} \cancel{l!} \binom{l}{l/2} (-1)^{l/2}$ $\Rightarrow l$ be even, $n = l/2$

$\Rightarrow \frac{d^l}{dx^l} \binom{l}{l/2} x^l (-1)^{l/2}$

$l! \binom{l}{l/2} (-1)^{l/2}$

$P_{2l}[0] = \frac{1}{4^l} \binom{2l}{l} (-1)^l$

$= \frac{1}{4^l} \frac{(2l)!}{l!(l!)}$

$\Rightarrow$

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$$P_{2l}[0] = \frac{1}{4^l} \frac{(2l)!}{l!\ l!} (-1)^l$$

$$= \frac{1}{4^l} \frac{(l+1)(l+2)\cdots(2l)}{l!} (-1)^l$$

$$\frac{d}{dx^l}(x^2-1)^l$$

$$(x^2-1)^l = \sum_{n=0}^{l} \binom{l}{n} (x^2)^n (-1)^{(l-n)}$$

$$\boxed{P_{2l}[0] = \frac{1}{4^l} \frac{(2l)!}{l!\,l!} (-1)^l} \quad \frac{d}{dx} \Rightarrow \text{differentiation gives the constant term}$$